On the Impact of Fundamentals, Liquidity and Coordination on Market Stability*

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Abstract

We develop a coordination game to model interactions between fundamentals and liquidity during unstable periods in financial markets. We then propose a flexible econometric framework for estimation of the model and analysis of its quantitative implications. The specific empirical application is carry trades in the yen–dollar market, including the turmoil of 1998. We find a generally very deep market, with low information disparities amongst agents. We observe occasionally episodes of market fragility, or turmoil with up by the escalator, down by the elevator patterns in prices. The key role of strategic behavior in the econometric model is also confirmed.

JEL: C13, C15, C22, C51, F31, G12, G15.

Keywords: global games, efficient method of moments, carry trades, tail risk, strategic behavior, financial crises

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1 Introduction

Market participants’ strategic behavior has the potential to fuel and magnify market turmoil and even trigger financial crises. To capture and understand such pathologies, we propose a global game theoretic model along with a flexible econometric framework for direct estimation using only publicly available data.

As we link global games and empirical finance, our contribution is twofold. First, from an empirical finance perspective, we develop a model with economically meaningful parameters that decomposes returns into fundamental and liquidity returns. Such a model can provide guidance for policy making and risk management. Second, from a global games perspective, we develop an econometric framework for estimation, analysis of quantitative implications and testing of data fit. Our empirical analysis indicates that global games are a sensible equilibria refinement.

Our specific empirical focus is on currency carry trades, i.e. the exploitation of violations of uncovered interest parity by buying high interest rate currencies and selling low interest rate currencies. In aggregate, the volume of carry trades can be significant, destabilizing currency markets. In a typical situation, a slow buildup of positions is followed by rapid unwinding once those positions are no longer so attractive. Therefore, the high interest rate currency rises slowly but then crashes suddenly, following what has been described as going up by the escalator – down by the elevator. While carry trades are most common in currency markets, they are also prevalent in markets such as fixed income and have been a prominent feature of the current global financial crisis.

Since the trading decisions of agents reinforce each other during such turbulent episodes, they are naturally modelled by coordination games theory such as the global games\(^1\) which provide our theoretic background. Most extant attempts of applying coordination games to financial data have focused on currency crises, assuming common knowledge of economic fundamentals in the spirit of models such as Obstfeld (1986). Common knowledge generally implies multiple equilibria and sunspots, as in the Markov–switching model of Jeanne and Masson (2000). In the absence of ad–hoc assumptions, multiple equilibria frustrates empirical implementations. However, by making the plausible assumption of asymmetric information, global games find a unique equilibrium.\(^2\)

\(^1\)See e.g. Morris and Shin (1998); Goldstein and Pauzner (2005); Morris and Shin (2004).

\(^2\)Several authors have estimated the role of information for the propensity for spec-
Our specific application is the extreme turbulence in the yen–dollar market in 1998 caused by the unwinding of carry trades. The specific events triggering this turbulence were unrelated to the Japanese market, such as the aftermath of the Russian and LTCM crisis as well as yield curve adjustments in the run-up to the adoption of the Euro. We use daily data from 1990 to 2004, enabling us to include both time periods with and without turmoil. We estimate the global game with the efficient method of moments proposed by Gallant and Tauchen (1996, 2002) and confirm the key role of strategic behavior in the econometric model.

We find this currency market to be generally very deep, i.e. the game players or strategic traders only rarely have a price impact and generate extra volatility. However, the estimated model also shows evidence of low information disparities amongst strategic traders. Their orders tend to be similar and generate market fragility, i.e. there are episodes of sharp changes in prices with small changes in fundamentals.

Our empirical application links market illiquidity to investors’ (lack of) risk appetite, itself is proxied by the Chicago Board Options Exchange (CBOE) Volatility Index (VIX), a widely used barometer of investor sentiment and market volatility. In this we are motivated by Cairns et al. (2007) and Brunnermeier et al. (2008) who find that an increase in the VIX is associated with the depreciation of currencies with high interest rates against currencies with low interest rates, and hence with carry trade losses.

We find significant evidence of asymmetry between the buy and sell sides of the market. Liquidity tensions on the sell side are much less frequent but also much more sensitive to risk appetite. In particular, strategic agents only have a positive price impact when the VIX is higher than 21%, while their negative price impact shows up for VIX values exceeding 37%.

This asymmetry translates into strong escalator–elevator effects in prices. Strategic traders create both positive trends and sharp corrections in prices. Their price impact component in returns has a positive mean, with strategic traders buying more frequently than selling, but generating large negative skewness, or tail risk.

Moreover, the estimated correlation between this component of returns and the component driven by fundamentals is positive, which shows that strategic traders amplify the movements in fundamentals. This is in line with the cumulative attacks using reduced form specifications motivated by global games, and find indirect empirical support for the theory. See inter alia Metz and Michaelis (2003), Prati and Sbracia (2002) and Tillmann (2004). More recently, Chen et al. (2008) study strategic complementarities with mutual fund data.
empirical literature that shows that financial crises are related to fundamen-
tals. See e.g. the papers cited in Chen et al. (2008) that relate failing banks
to bad fundamentals.

The rest of the paper is organized as follows. We develop the global game
to model returns under liquidity tensions in Section 2. Then we construct
an econometric model of fundamentals and liquidity in Section 3. In Section
4, we apply our model to yen-dollar data that covers the market turmoil in
1998. Finally, we present our conclusions in Section 5. Proofs are relegated
to an appendix.

2 Returns under Liquidity Tensions

Consider a market for a financial asset, with two categories of agents, strate-
gic agents and nonstrategic agents. The strategic agents trade for short term
speculative reasons, could be proprietary traders or hedge funds, and play a
coordination game. The nonstrategic agents are long–term investors provid-
ing residual demand/supply.

2.1 Return Decomposition

The strategic agents are uniformly distributed on the interval [0, 1] and each
agent makes a binary decision to buy or sell one unit of the asset. The
fraction of agents selling is \(\lambda_t \in [0, 1]\).

The observed return on the asset during trading round \(t\), \(r_t\), can be decom-
posed into two components:

\[
\text{Observed return } (r_t) = \text{fundamental return } (v_t) + \text{liquidity return } (\ell_t).
\]

(1)

The fundamental return \(v_t\) is driven by asset fundamentals (the present value
of the asset’s expected future dividends) capturing the exogenous arrival of
information during the period. The fundamental return is assumed to be
normally distributed conditional on mean \(m_t\) and precision \(\alpha_t\)

\[
v_t \sim N(m_t, \alpha_t^{-1}) .
\]

(2)

The liquidity return \(\ell_t\) represents the price impact of strategic agents

\[
\ell_t = c_t^+ (1 - \lambda_t) - c_t^- \lambda_t ,
\]
where \( c_t^- \geq 0 \) and \( c_t^+ \geq 0 \). \((c_t^-, c_t^+)\) is a measure of the (lack of) liquidity or market depth on the sell and buy side, respectively, driven by the willingness of nonstrategic agents to absorb the orders from strategic agents. The higher \((c_t^-, c_t^+)\) is, the higher the price concession required by the nonstrategic traders. In the absence of liquidity tensions, the situation expected most of the time, \( c_t^- = c_t^+ = 0 \).

Price changes can be expressed as

\[
p_t - p_{t-1} = r_t = v_t + \ell_t, \tag{3}
\]

so that the current price \( p_t \) is equal to some initial price \( p_0 \) plus the accumulation of fundamental and liquidity returns

\[
p_t = p_0 + \sum_{s=1}^{t} \left( v_s - c_s^- \lambda_s + c_s^+ (1 - \lambda_s) \right).
\]

This representation of prices follows the notion of liquidity in Grossman and Miller (1988), as well as Bernardo and Welch (2004), Morris and Shin (2004) and Brunnermeier and Pedersen (2005).

In most standard one period models, either \( c_t^+ \) or \( c_t^- \) is zero, since such models solely focus on a single direction impact, e.g. a decision to launch a speculative attack or not. Given that our ultimate objective is to estimate the model from time series data, we need to model both buying and selling decisions as well as their price impacts.

### 2.2 Strategic Agents

The strategic agents are short–term speculators that care about the value of their positions at the end of each trading round and expect to profit during liquidity tensions by exploiting a simple trading rule: Agent \( i \) buys if she expects a positive return\(^3\) by the end of the turbulent period, i.e. if

\[
E \left( r_t \mid \Omega^i_t \right) > 0 \tag{4}
\]

where \( \Omega^i_t \) means information available at the beginning of period, and sells otherwise. The fundamental return, as well as the trading decisions of other strategic agents, are not observed prior to trading.

\(^3\)To simplify the expressions, we abstract from dividends and financing costs, which are relevant in carry trades. If we added these features then the relevant (excess) return for the strategic trader would be \( (p_t - p_{t-1}) + d_t \), where \( d_t \) is the corresponding asset income minus the corresponding financing cost.
This trading decision rule captures the coordinated behavior of strategic agents during periods of liquidity tensions. Given an expected $v_t$, if agent $i$ expects a sufficiently high $\lambda_t$ she will sell, while buying if expecting a $\lambda_t$ that is low enough. Even if the price impact of an individual strategic agent is negligible, they exert a significant price impact if a sufficient number of them make the same trading decision during times of illiquidity.\footnote{More detailed theoretical models can be found in Bernardo and Welch (2004) and Morris and Shin (2004), who model sales coordination under strong liquidity tensions and a random execution of orders. The former authors follow the spirit of bank-run models, while the latter authors rely on loss limits. Brunnermeier and Pedersen (2005), and Plantin and Shin (2007), show coordination in dynamic models without relying on a random execution of orders. The former authors focus on predatory trading, while the latter authors introduce funding externalities in the spirit of Brunnermeier and Pedersen (2007).}

The strategic agents only have access to incomplete information regarding the residual demand they face. They know the price impacts, $(c^+_t, c^-_t)$ and the parameters of the fundamental returns distribution $(m_t, \alpha_t)$. They do not, however, observe $v_t$ itself, which in turn depends on the arrival of information during the period, before their trading decision. Each strategic agent $i$ receives a private signal $x^i_t$:

$$x^i_t = v_t + \varepsilon^i_t, \quad \varepsilon^i_t \sim N \left(0, \beta_t^{-1}\right),$$

where the signal noise $\varepsilon^i_t$ is independently and identically distributed across agents. The parameter $\beta_t$ is the signal precision which is known by all agents and measures information disparities among strategic agents.

Hence the relevant information set of each agent $i$ for making a trading decision is:

$$\Omega^i_t = \left(m^i_t, \alpha_t, \beta_t, c^+_t, c^-_t\right),$$

where $m^i_t$ is the posterior mean of $v_t$ for this agent after observing $x^i_t$.

### 2.3 Market Equilibrium

Consider monotone strategies where a strategic agent buys if her private signal (given her private information $\Omega^o_t$) exceeds $x^o_t$, and sells otherwise. Applying a law of large numbers to the cross section of strategic agents, the equilibrium fraction of agents selling is

$$\lambda^e_t = \Pr \left(x_t < x^o_t \mid \Omega^o_t, v_t\right),$$

where $x_t$ represents the distribution of signals across agents. The following Lemma characterizes $\lambda^e_t$.\footnote{More detailed theoretical models can be found in Bernardo and Welch (2004) and Morris and Shin (2004), who model sales coordination under strong liquidity tensions and a random execution of orders. The former authors follow the spirit of bank-run models, while the latter authors rely on loss limits. Brunnermeier and Pedersen (2005), and Plantin and Shin (2007), show coordination in dynamic models without relying on a random execution of orders. The former authors focus on predatory trading, while the latter authors introduce funding externalities in the spirit of Brunnermeier and Pedersen (2007).}
Lemma 1 There is a unique equilibrium in the coordination game if and only if
\[
\frac{\alpha_t^2 (\alpha_t + \beta_t)}{\beta_t (\alpha_t + 2\beta_t)} \leq \frac{2\pi}{(c_t^+ - c_t^-)^2}, \tag{7}
\]
with
\[
\lambda_t^e = \Phi \left[ \sqrt{\beta_t} (x_t^0 - v_t) \right],
\]
where \(x_t^0\) is the signal of the strategic agent that is indifferent between buying and selling.

\(\Phi (\cdot)\) and \(\phi (\cdot)\) are the cumulative distribution function and density of the standard normal distribution, respectively. The proof and the characterization of \(x_t^0\) are shown in the Appendix. The uniqueness condition is a lower bound on \(\beta_t\) for given \(\alpha_t\) and price impacts. That is, a unique equilibrium requires private information to be precise enough with respect to public information. Note that \(\lambda_t^e\) is monotonically decreasing in \(v_t\), providing a clear connection between fundamentals and equilibrium. Moreover, the price impact of strategic agents amplifies the effect of \(v_t\) on \(r_t\) since Lemma 1 implies that the observed return in equilibrium is
\[
r_t = (v_t + c_t^+) - (c_t^- + c_t^+) \Phi \left[ \sqrt{\beta_t} (x_t^0 - v_t) \right], \tag{8}
\]
and hence, under liquidity tensions,
\[
\frac{\partial r_t}{\partial v_t} = 1 + (c_t^- + c_t^+) \sqrt{\beta_t} \phi \left[ \sqrt{\beta_t} (x_t^0 - v_t) \right] > 1.
\]

While observed returns are decomposed into fundamental and liquidity returns in (1), \(\lambda_t^e\) implies a complex interaction of fundamentals and liquidity parameters \((m_t, \alpha_t, \beta_t, c_t^+, c_t^-)\) in the definition of \(r_t\). To clarify this, consider the special case where \(\beta_t \to \infty\), i.e. the signal precision is unbounded and hence there are small information disparities. In this case, the marginal agent’s beliefs about \(\lambda_t^e\) converge to being uniform in \([0, 1]\) whatever distribution of the signals. Even though fundamental uncertainty becomes negligible as \(\beta_t \to \infty\), the marginal agent expects half of the agents to sell and consequently her signal is \(x_t^0 = 0.5 (c_t^- - c_t^+)\). We arrive at the following corollary:

**Corollary 2** If \(\beta_t \to \infty\) there is a unique equilibrium in the coordination game with
\[
\lambda_t^e = I \left[ v_t < 0.5 \left( c_t^- - c_t^+ \right) \right],
\]
where \(I [\cdot]\) denotes the indicator function.
\( \lambda_t^e \) is not a continuous function of \( v_t \) and hence we find market fragility where small changes of \( v_t \) around 0.5 \((c_t^- - c_t^+)\) produce a sharp change in \( \lambda_t^e \). In addition, if liquidity tensions on the sell side are much higher than on the buy side, \( c_t^- >> c_t^+ \), then even a relatively high \( v_t \) might not be enough to avoid \( \lambda_t^e = 1 \). And vice versa, if liquidity tensions on the buy side are much higher than on the sell side \( c_t^+ >> c_t^- \) then even a relatively low \( v_t \) might not be enough to avoid \( \lambda_t^e = 0 \).

The corollary shows that the observed return in equilibrium is

\[
 r_t = (v_t + c_t^+) - (c_t^- + c_t^+) I \left[ v_t < 0.5 \left( c_t^- - c_t^+ \right) \right]
\]

under unbounded precision.\(^5\)

This simple expression of \( \lambda_t^e \) also helps in clarifying the differences and similarities between our global game, where asymmetric information among the strategic agents helps to pin down a unique equilibrium, and a coordination game where \( v_t \) is common knowledge. If fundamental returns are low enough, i.e. \( v_t < -c_t^+ \), then both our global game and the common knowledge game give \( \lambda_t^e = 1 \). Similarly, if fundamental returns are high enough, i.e. \( v_t > c_t^- \), then both games give \( \lambda_t^e = 0 \).

The two outcomes only (may) differ at intermediate values of the fundamental returns. In that region, the global game pins down a unique \( \lambda_t^e \) depending on \( v_t \) being to the left or right of 0.5 \((c_t^- - c_t^+)\), while both \( \lambda_t^e = 0 \) or 1 are possible outcomes in the common knowledge counterpart. Therefore, to take the latter model to the data, we would need to define an ad–hoc equilibrium selection procedure. Nevertheless, two standard refinements of the common knowledge equilibria coincide with the global game equilibrium in (9). The risk–dominant equilibrium is known to coincide with the unique equilibrium in a global game with unbounded signal’s precision. That is not necessarily the case for the payoff–dominant equilibrium, but it coincides with the global game outcome in our model.

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\(^5\)The introduction of dividends and financing costs only requires two changes in the previous results. If \( d_t \) denotes the asset income minus the corresponding financing cost then the corollary changes to \( \lambda_t^e = I \left[ v_t + d_t < 0.5 \left( c_t^- - c_t^+ \right) \right] \). Regarding the lemma, we should take into account \( d_t \) in the definition of \( m_t^e \) in the Appendix.
3 An Econometric Model of Fundamentals and Liquidity

The model developed above depends on the state variables \((m_t, \alpha_t, \beta_t, \sigma_t^+, \sigma_t^-)\). It is unrealistic to expect those state variables to remain constant over time, and for the purpose of estimation we need to model their dynamic evolution. In this our approach is reminiscent of Foster and Viswanathan (1995) in their empirical application of Kyle (1985).

3.1 Fundamental Returns

The distribution of fundamental returns is characterized in (2) by its mean and precision \((m_t, \alpha_t)\). The fundamental mean can be linked to a predictor of fundamentals that is observable by strategic agents before they take trading decisions, i.e.

\[
m_t = m(f_t)
\]

for some function \(m(\cdot)\). The choice of both \(f_t\) and \(m(\cdot)\) will depend on the particular empirical application.

In addition, we need to model the volatility of fundamental returns. In this we are guided by both stylized facts of financial returns and theoretical considerations. In particular, financial returns are known to exhibit volatility clusters and have fat tails. This of course does leave us with a broad range of models, typically either of the the GARCH or stochastic volatility\(^6\) (SV) type. Of those, the SV models are more natural, because of their tighter connections with decision–making, see e.g Tauchen and Pitts (1983).

Following a standard SV model, the fundamental return precision \(\alpha_t^{-1}\) is a stationary AR(1) process in logs with normal innovations:

\[
-\ln \alpha_t - \gamma_1 = \gamma_2 (-\ln \alpha_{t-1} - \gamma_1) + \gamma_3 u_t, \quad u_t \overset{iid}{\sim} N(0, 1), \quad |\gamma_2| < 1, \quad (10)
\]

where \(u_t\) and the Gaussian innovation of \(v_t\) are independent to be consistent with the assumed information structure in trading decisions. This specification nests cases such as a deterministic \(\alpha_t\) with \(\gamma_3 = 0\) or lack of persistence in \(\alpha_t\) with \(\gamma_2 = 0\).

\(^6\)Proposed originally by Clark (1973) and Taylor (1986).
3.2 Private Signals

Another relevant dimension of fundamental returns is the extent to which they are revealed to the strategic agents through the signal distribution (5) with the precision $\beta_t$. A consequence of (7) in Lemma 1 is that the uniqueness condition needs to be satisfied in each period with liquidity tensions in the econometric model because $(\alpha_t, \beta_t)$ and $(c^+_t, c^-_t)$ change over time. Therefore, the stochastic process of $\beta_t$ could be expressed as

$$\beta_t = \beta(b_t)$$

for some function $\beta(\cdot)$ that satisfies the lower bound denoted by $b_t$. A simple example of such a function is:

$$\beta_t = b_t + \eta, \quad (11)$$

where $\eta \geq 0$ is a real number that defines $\beta_t$ given $b_t$, i.e. conditional on $(\alpha_t, c^+_t, c^-_t)$. This parameter is a measure of the relative precision of strategic agents’ private information about fundamentals at the end of period with respect to their public information. If $\eta$ is very low we find the lowest level of private signals’ precision that is compatible with the uniqueness condition (7). If $\eta$ is very high then private information is much more reliable than public information and signals are so precise about $v_t$ that there are low information disparities across agents. The higher its value the closer we are to the equilibrium described in Corollary 2.

3.3 Market (Il)liquidity

Liquidity tensions on the buy and sell side are represented by $(c^+_t, c^-_t)$ in (3). Several measures of liquidity have been proposed, e.g. Danielsson and Payne (2002), Amihud et al. (2005) and Pastor and Stambaugh (2003), some of which depend on proprietary data. The specific market structure dictates the specific modelling of price impacts and choice of explanatory variables. For example, if the underlying market structure is composed by market makers as on the New York Stock Exchange (NYSE), it would be natural to use tick–by–tick observations from that exchange, the so–called Trade and Quote (TAQ) data.

As a general modelling approach, we link $(c^+_t, c^-_t)$ to a proxy of the willingness of nonstrategic agents to absorb the orders from strategic agents. Such a
willingness, or risk appetite, depends on their risk aversion and risk itself.\footnote{The connection of liquidity and risk has a long tradition in finance, see e.g. Benston and Hagerman (1974).} Of the various publicly available measures of (inverse) risk appetite, the Chicago Board of Exchange volatility index (VIX) may be the most widely used one in practice, and what we use here.

Both price impacts are a function of the inverse risk appetite denoted by $\sigma_t$,

$$c^+_t = c^+(\sigma_t), \quad c^-_t = c^-(\sigma_t),$$

for some functions $c^+(\cdot)$ and $c^-(\cdot)$. In addition, to take this model to the data, we need a specific functional form. This function should be such that there are no liquidity tensions when $\sigma_t$ is low and, when $\sigma_t$ is high, liquidity tensions are present and increasing along with $\sigma_t$.

A natural specification is a linear specification capturing both the presence and magnitude of liquidity tensions as a function of $\sigma_t$, separately for each side of the market. In particular:

$$c^+_t = \max\left\{\phi^+_1 + \phi^+_2 \sigma_t, 0\right\}, \quad c^-_t = \max\left\{\phi^-_1 + \phi^-_2 \sigma_t, 0\right\}. \quad (12)$$

In this example, the parameters $\phi^+_2$ and $\phi^-_2$ measure the sensitivity of market liquidity to risk appetite, while $-\phi^+_1/\phi^+_2$ and $-\phi^-_1/\phi^-_2$ define $\sigma_t$ thresholds for the presence of liquidity tensions.

## 4 Empirical Application

The relevance of our methodology ultimately rests on its ability to capture important tumultuous events in financial markets. Our empirical application is based on a particular scenario of market instability driven by carry trades. One of the best known examples of a carry trade induced market turmoil occurred in 1998, when over two days in October — 7th and 8th — the dollar fell from 131 yen to 112 yen by lunchtime in London on Thursday the 8th, bouncing back sharply to end New York trading at 119 yen. October 7th and 8th 1998 were two of the most turbulent days of currency trading in financial markets. These events are discussed e.g. by Morris and Shin (2000) and Danielsson and Shin (2003). In the week beginning October 5th, the decline of the dollar against the yen accelerated sharply — closing down roughly 15% over the week consistent with the rapid unwinding of the yen carry trades. Global events such as the Russian default, LTCM, and the yield
curve adjustments leading to the Euro initiated the unwinding of previous long positions in dollars.

We use daily yen-dollar exchange rates as reported by the New York Fed and estimate the model using continuously compounded returns in percentage terms as $r_t$ from (3). While our main event of interest occurs in 1998, the entire data sample used for estimation spans 1990 to 2004. Since our model addresses both liquid and illiquid time periods, the data set needs to be sufficiently long so that we obtain a good estimation of the underlying processes of fundamental and liquidity returns. At the same time we avoid an arbitrary choice of the time period to be fitted by the model. We show the exchange rates in levels and continuously compounded returns in Figures 1 and 2 respectively, whilst descriptive statistics of the returns can be found in Table 1.

Figure 1 shows clear escalator–elevator effects in 1998 characteristic of carry trade induced financial turmoil. This is visible in the returns in Figure 2 as a particularly pronounced volatility cluster. Furthermore, this translates into the negative skewness in returns shown in Table 1.
The fundamental mean $m_t$ is driven by the interest rate spread between the US and Japan, since it can be interpreted as the dividend of holding dollars against yen.\(^8\) Indicate $i^{\text{JAP}}_t$ and $i^{\text{US}}_t$ as the daily rate of the monthly\(^9\) London interbank offered rate (LIBOR) in yens and dollars, respectively. This data was obtained directly from [www.bba.org](http://www.bba.org). The interest rate spread is calculated by $100 \log \left( \frac{(1 + i^{\text{JAP}}_{t-1})}{(1 + i^{\text{US}}_{t-1})} \right)$ and the fundamental mean $m_t$ is equal to that spread measured on a daily basis (divided by 250). In this way, we follow the spirit of the uncovered interest rate parity\(^10\) in the sense of an expected (fundamental) dollar appreciation, $m_t > 0$, when $i^{\text{JAP}}_{t-1} > i^{\text{US}}_{t-1}$.

We show the spread in Figure 3, and its descriptive statistics in Table 1. This series is quite persistent and, during most part of the sample period, the yen

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\(^8\)Other macro variables such as real output and nominal money supply are not available at the daily frequency.

\(^9\)We should use overnight rates but the corresponding time series starts in 2001. Fortunately, the behavior of overnight rates is closely related to monthly rates. For the period of time were both are available, the correlation is 0.975 and OLS of overnight rates onto one-month rates gives an intercept of $-0.001$ and a slope of 0.964.

\(^10\)In spite of its theoretical appealing, it is well known that the uncovered interest rate parity is at odds with empirical evidence. See for instance Evans and Lyons (2002).
Table 1: Data summary statistics 1990–2004

FX return is the daily continuously compounded return of the yen/dollar exchange as in Figure 2. Spread is the annualized log-interest rate differential between the yen and the dollar as in Figure 3. VIX is the annualized option implied volatility index from the CBOE as in Figure 4. All variables in %.

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>s.d.</th>
<th>skewness</th>
<th>kurtosis</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>FX return</td>
<td>-0.009</td>
<td>0.704</td>
<td>-0.501</td>
<td>3.954</td>
<td>-5.630</td>
<td>3.240</td>
</tr>
<tr>
<td>Spread</td>
<td>-2.576</td>
<td>2.368</td>
<td>0.183</td>
<td>-1.309</td>
<td>-6.369</td>
<td>2.152</td>
</tr>
<tr>
<td>VIX</td>
<td>19.93</td>
<td>6.352</td>
<td>0.910</td>
<td>0.742</td>
<td>9.31</td>
<td>45.74</td>
</tr>
</tbody>
</table>

interest rate is lower than the dollar interest rate.

Figure 4 shows the evolution of the VIX index while Table 1 shows the corresponding descriptive statistics. We can see that the VIX peaks at financial turmoil such as the Russian default of August 1998, the terrorist attacks of September 2001, and the WorldCom accounting scandal and bankruptcy plus other factors (including geopolitical tensions) in June–July 2002.

4.1 Model Estimation

Due to the nonlinear dependence on non-normal dynamic latent variables our model cannot be directly estimated by either maximum likelihood or the generalized method of moments. However, simulation methods have successfully been applied to SV models and we follow that approach here, in particular the efficient method of moments (EMM) proposed by Gallant and Tauchen (1996, 2002).

The EMM approach is based on first estimating an auxiliary model whose purpose is to provide optimal moments conditions. We then simulate the theoretical model for given parameter values and minimize the distance between the moments from the simulated data and the observed data. Given the large number of local minima characteristic of such estimation, we follow the Markov chain Monte Carlo procedure of Chernozhukov and Hong (2003).

The auxiliary model provides a score generator by using a semi nonparametric (SNP) method as described e.g. in Gallant and Tauchen (2006). The SNP density is based on the notion that a Hermite expansion can be used to provide a general purpose approximation of the density function of the
data of interest, with a parametric model capturing dependence in the first and second moments. In our case we get an AR(1)–GARCH(1,1) model with a semi-parametric density for innovations based on a fourth order Hermite polynomial.\textsuperscript{11} This translates into 14 auxiliary parameters.

We can expect EMM to be nearly fully efficient (like the maximum likelihood estimator) when SNP is used as the score generator since it is a flexible reduced form model that closely approximates the actual distribution of the data. This implementation of the EMM procedure selects the optimal testable properties of the model. Therefore, the value of the criterion function provides a good test of the adequacy of a theoretical model regarding the dynamic and steady state properties of data.

We initially estimated \( \eta \) in (11) along with the other model parameters, i.e. we estimated the return model in (8). However the results indicated unbounded signals precision (\( \beta_t \to \infty \)) in the sense that \( \eta \) was so high that \( \lambda_{et} \) in Lemma 1 was actually equivalent to \( \lambda_{et}^c \) in Corollary 2. This means low information disparities across strategic agents, which is a natural feature of FX markets. In particular, the estimated \( \hat{\eta} \) was 3484 and implied a process \( \beta_t \) with a similar average, while the estimated process \( \alpha_t \) had an average lower than 5, i.e. we found \( \beta_t >> \alpha_t \). The corresponding process \( \lambda_{et}^c \) was outside the interval \([0.001, 0.999]\) more than 90% of the time.

Consequently, the estimates in the remainder of this section refer to model (9), where we impose \( \beta_t \to \infty \). This model can equivalently be derived from two standard refinements in the corresponding common knowledge game (the payoff and risk–dominant equilibria). The estimates of the model parameters \( (\gamma_1, \gamma_2, \gamma_3, \phi_1^+, \phi_2^+, \phi_1^-, \phi_2^-) \) can be found in Table 2. The model is not rejected, with the data fit very good. The criterion function is 8.95, with an associated p–value of 25.6\%, and all parameters significant at the 5\% level except \( \phi_1^- \).

The SV component of fundamental returns shows an average level of \( \hat{\gamma}_1 = -1.32 \) in logs, persistence \( \hat{\gamma}_2 = 0.34 \) and volatility of volatility \( \hat{\gamma}_3 = 0.66 \). The main liquidity components on the buy side are \( \hat{\phi}_1^+ = -0.04 \) and \( \hat{\phi}_2^+ = 0.23 \), while the sell estimates are \( \hat{\phi}_1^- = -9.34 \) and \( \hat{\phi}_2^- = 3.45 \). We find that high VIX, as a proxy for nonstrategic agents’ lack of risk appetite, is related to low liquidity for the yen carry trades.

The illiquidity thresholds are \( -\hat{\phi}_1^+ / \hat{\phi}_2^- = 0.153 \) and \( -\hat{\phi}_1^- / \hat{\phi}_2^- = 2.707 \). That is, illiquidity tensions on the sell side \( \epsilon_t^- \) are much less frequent but also much more sensitive to risk appetite. Since the estimates with VIX use it in

\textsuperscript{11}Specifically, SNP models are described by 8 digits and our selected model is 11114010. The first four digits describe the AR-GARCH structure, while the remaining four digits describe the Hermite polynomial. See Gallant and Tauchen (2006) for additional details.
Table 2: Estimation results

The process $\alpha_t^{-1}$ is defined in (10), while the processes $(c^+_t, c^-_t)$ are defined in (12). The standard errors are shown in parenthesis under each estimate. The number of simulated returns is 100,000.

<table>
<thead>
<tr>
<th>SV parameters</th>
<th>liquidity parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_t^{-1}$</td>
<td>$c^+_t$</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>$\gamma_2$</td>
</tr>
<tr>
<td>-1.315</td>
<td>0.336</td>
</tr>
<tr>
<td>(0.111)</td>
<td>(0.116)</td>
</tr>
</tbody>
</table>

standardized terms, the results imply that liquidity tensions are present on the buy side when the VIX exceeds 20.9% and 37.1% on the sell side. Both sides of the market are usually infinitely deep, i.e. strategic agents affect the market only rarely, 1.6% of the time on the sell side, and 38.3% of the time on the buy side.

Table 3 shows the relative magnitude of $c^+_t$ and $c^-_t$ above thresholds of increasing size as multiples of the volatility of the yen–dollar exchange rate returns. The asymmetry between the buy and sell sides is clear, $c^+_t$ is characterized by a high frequency of low values while $c^-_t$ is characterized by a low frequency of high values. This is consistent with the up by escalator–down by elevator scenario.

For comparison, we also estimated a descriptive statistical model with stochastic volatility and leverage effect (LSV), i.e. with correlations between shocks to returns and shocks to volatility. This effect can be interpreted as a descriptive modelling of asymmetric tail events, as opposed to our explicit liquidity mechanism. For instance, Gagnon and Chaboud (2007) report high actual and implied volatility around the sharp yen appreciation in October 1998.

The LSV model can be described simply as $r_t \sim N(m_t, \alpha_t^{-1})$, where both $m_t$ and $\alpha_t$ follow the same processes as in our model, except for the correlation between shocks to $r_t$ and shocks to $-\ln \alpha_t$. The correlation estimate is $-0.14$ to approximate the up by escalator–down by elevator features in the data. The estimates of the counterparts of $(\gamma_1, \gamma_2, \gamma_3)$ are $(-1.02, 0.64, 0.58)$. The presence of liquidity tensions decreases the level and persistence of stochastic volatility with respect to the LSV model.

The criterion function is 12.76, with an associated p–value of 23.7%. That is,
Table 3: Asymmetry in market illiquidity

Relative frequency and average magnitude of price impacts above thresholds of increasing size as multiples of the volatility of the yen-dollar exchange rate returns, $\sigma_{FX}$, i.e. $c_i^+, c_i^- > i \times \sigma_{FX}, i = 0, 1, \ldots, 5$.

<table>
<thead>
<tr>
<th>magnitude $i \times \sigma_{FX}$</th>
<th>$c_i^-$ frequency(%)</th>
<th>mean</th>
<th>$c_i^+$ frequency(%)</th>
<th>mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.61</td>
<td>1.77</td>
<td>38.28</td>
<td>0.20</td>
</tr>
<tr>
<td>1</td>
<td>1.26</td>
<td>2.18</td>
<td>0.70</td>
<td>0.78</td>
</tr>
<tr>
<td>2</td>
<td>0.81</td>
<td>2.81</td>
<td>0</td>
<td>—</td>
</tr>
<tr>
<td>3</td>
<td>0.59</td>
<td>3.18</td>
<td>0</td>
<td>—</td>
</tr>
<tr>
<td>4</td>
<td>0.35</td>
<td>3.70</td>
<td>0</td>
<td>—</td>
</tr>
<tr>
<td>5</td>
<td>0.19</td>
<td>4.10</td>
<td>0</td>
<td>—</td>
</tr>
</tbody>
</table>

our model does not perform worse than a LSV.\footnote{Without a leverage effect, the SV model is rejected by the data. A simple stochastic volatility model is not suitable to explain the yen–dollar exchange rate.} The data fit of our model is slightly better, but the criterion function p–values are similar because our model relies on more parameters.

4.2 Model Properties

We further illustrate the estimation results by means of simulations.\footnote{All the computations in this section refer to simulations of 30 times the sample size, which means more than 100,000 simulated returns.} We simulate the fundamental return precision $\alpha_t$ from (10) and compute the fundamental return mean $m_t$ from interest rates. Given those two processes, we simulate the fundamental return $v_t$ from (2). Finally, we simulate the total return $r_t$ from (9), which requires the fundamental return $v_t$ and the VIX–driven $(c_t^+, c_t^-)$ from (12).

First, we explore the impact of coordination and liquidity in returns by comparing the different statistical properties of total returns and their fundamental and liquidity components. Table 4 shows some sample statistics for the observed and simulated data.

The model parameters were estimated by using the optimal moments selected by the EMM procedure. Given the low EMM criterion function, it is not
Table 4: Static properties

Mean, standard deviation, coefficient of skewness, and coefficient of excess kurtosis of observed returns and simulated returns under estimates in Table 2. Simulated returns are decomposed as in (1).

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>s.d.</th>
<th>skewness</th>
<th>kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>data</td>
<td>-0.009</td>
<td>0.704</td>
<td>-0.501</td>
<td>3.954</td>
</tr>
<tr>
<td>$r_t$</td>
<td>0.007</td>
<td>0.680</td>
<td>-0.738</td>
<td>4.861</td>
</tr>
<tr>
<td>$v_t$</td>
<td>-0.010</td>
<td>0.585</td>
<td>-0.001</td>
<td>1.788</td>
</tr>
<tr>
<td>$\ell_t$</td>
<td>0.018</td>
<td>0.291</td>
<td>-9.444</td>
<td>116.6</td>
</tr>
</tbody>
</table>

It is surprising that the four statistics of the simulated data are close to the actual data. Ultimately this supports the conclusion that our model has successfully captured the stylized statistical facts of the yen–dollar exchange rate.

In addition, we can compare those four moments across the different components of asset returns. The liquidity component adds a positive mean but a quite negative skewness, or tail risk, to fundamental returns. In fact, we find $\lambda^e_t = 1$ in 44% of periods with liquidity tensions, i.e. strategic traders sell less than half the time they have a significant price impact. Similarly, this component has a low volatility but an extremely high kurtosis compared to fundamental returns.

The theoretical model shows that ceteris paribus liquidity returns amplify the effect of fundamental returns. Obviously, $(c^+_t, c^-_t)$ varies over time as $v_t$ does. We can quantify the comovement between fundamental and liquidity returns by their time series correlation, which is 11% in our estimated model. Similarly, the correlation between fundamental returns and $\lambda^e_t$ itself is -74%.

The global game, and equivalently the risk–dominant and payoff–dominant equilibrium of the corresponding common knowledge game, induce a strong negative correlation between sales and fundamental returns.

We can also study the dynamic properties of the data, in particular the autocorrelation in levels and squares, presented in Table 5. The observed returns exhibit low, but not zero autocorrelation, similar to our model. Note that this is the result of the interaction between a high autocorrelation in liquidity returns and the lack of autocorrelation in fundamental returns.

Our model also captures the higher autocorrelation in the squared returns which is driven again by the interaction of fundamental and liquidity returns.
Table 5: Dynamic properties

Autocorrelations of observed returns and simulated returns under estimates in Table 2. Autocorrelations are computed for the first five lags, first for the returns and then for squared returns.

<table>
<thead>
<tr>
<th>lags</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>returns</td>
<td>0.025</td>
<td>0.013</td>
<td>-0.043</td>
<td>0.015</td>
<td>0.007</td>
</tr>
<tr>
<td>$r_t$</td>
<td>0.096</td>
<td>0.051</td>
<td>0.046</td>
<td>0.037</td>
<td>0.021</td>
</tr>
<tr>
<td>$v_t$</td>
<td>0.002</td>
<td>0.000</td>
<td>0.004</td>
<td>-0.003</td>
<td>0.000</td>
</tr>
<tr>
<td>$\ell_t$</td>
<td>0.529</td>
<td>0.274</td>
<td>0.224</td>
<td>0.212</td>
<td>0.105</td>
</tr>
<tr>
<td>squared returns</td>
<td>0.211</td>
<td>0.119</td>
<td>0.102</td>
<td>0.069</td>
<td>0.097</td>
</tr>
<tr>
<td>$r_t$</td>
<td>0.348</td>
<td>0.191</td>
<td>0.206</td>
<td>0.196</td>
<td>0.132</td>
</tr>
<tr>
<td>$v_t$</td>
<td>0.039</td>
<td>0.014</td>
<td>0.005</td>
<td>0.003</td>
<td>0.001</td>
</tr>
<tr>
<td>$\ell_t$</td>
<td>0.503</td>
<td>0.237</td>
<td>0.269</td>
<td>0.262</td>
<td>0.164</td>
</tr>
</tbody>
</table>

The former show autocorrelation much lower than the data, while the latter show autocorrelation much higher than the data.

A detailed analysis of the escalator–elevator effect is presented in Table 6, which shows the frequency of returns higher than their mean plus a multiple of their standard deviation, and returns lower than their mean minus a multiple of their standard deviation. The asymmetry in tail events in the data is clear, with a higher frequency of strong dollar depreciations vs. strong appreciations. This asymmetry is well captured by the coordination model, in particular the liquidity returns.

We further illustrate the properties of the model by plotting typical sample paths from the simulations, over 2,000 days. Figures 5(a) and 5(b) show the time series of the two return components, fundamental and liquidity returns, while their sum is shown in Figure 5(c), and the cumulative returns or prices (with an initial price of 1) in Figure 5(d).

Notice the different behavior of both return components. The impact of strategic traders is given by the liquidity returns. They are zero for long periods of time but become positive and low for a part of the time, and very negative for a small number of periods. This means we can see a dramatic escalator–elevator episode, or the realization of tail risk, in the prices of Fig-
Table 6: Escalator–elevator effect in returns

Frequency in % of returns higher than their mean plus a multiple of their standard deviation (+ row), and lower than their mean minus a multiple of their standard deviation (- row). Each column refers to a different multiple. Computations under the estimates in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>data</td>
<td>50.24</td>
<td>12.15</td>
<td>2.10</td>
<td>0.43</td>
<td>0.13</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>49.76</td>
<td>11.94</td>
<td>3.18</td>
<td>0.99</td>
<td>0.32</td>
<td>0.08</td>
</tr>
<tr>
<td>( r_t )</td>
<td>51.36</td>
<td>12.64</td>
<td>1.99</td>
<td>0.33</td>
<td>0.06</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>48.64</td>
<td>11.85</td>
<td>2.52</td>
<td>0.86</td>
<td>0.45</td>
<td>0.24</td>
</tr>
<tr>
<td>( v_t )</td>
<td>50.06</td>
<td>13.45</td>
<td>2.64</td>
<td>0.54</td>
<td>0.11</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>49.94</td>
<td>13.41</td>
<td>2.66</td>
<td>0.52</td>
<td>0.11</td>
<td>0.02</td>
</tr>
<tr>
<td>( \ell_t )</td>
<td>20.00</td>
<td>4.99</td>
<td>0.30</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>80.00</td>
<td>1.13</td>
<td>1.08</td>
<td>0.98</td>
<td>0.87</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Figure 5(d). Strategic traders create both positive trends and sharp corrections in prices.

4.3 Robustness of Results

The model shows a good data fit but it remains to be demonstrated whether the empirical results are driven by the strategic behavior of agents, or simply the additional econometric structure of fundamentals and liquidity. To investigate this, we study some alternative proxies and specifications.\(^{14}\)

The use of the interest rate differential and stochastic volatility with fundamental returns seems plausible and general enough. Therefore our robustness checks will focus on market illiquidity. The empirical evidence below suggests that our estimation results are driven jointly by the economic and statistical structure of the model, with the strategic component of the model essential.

First, \( \sigma_t \) can be changed. As an alternative proxy of global volatility in financial markets, we used a volatility forecast from the Morgan Stanley Capital International (MSCI) index. We found similar parameter estimates and model properties in general. This is not surprising because the correla-

\(^{14}\)The computations of this section are available upon request from the authors.
Figure 5: Simulated outcomes
tion between the new proxy and the VIX is around 0.4. In particular, the estimated $\gamma_2$ is slightly higher and closer to its estimate in the LSV model, due to the the autocorrelation in the new proxy being lower than in the VIX.

Second, we can study different specifications of market illiquidity as a function of $\sigma_t$. We do not look for the best specification in terms of data fitting, but simply study one alternative specification where there is not a clear cut distinction between liquid and illiquid periods. That is, strategic traders may have a price impact, albeit very small, every period. This can be easily achieved by

$$c^+_t = \exp \left( \phi^+_1 + \phi^+_2 \sigma_t \right), \quad c^-_t = \exp \left( \phi^-_1 + \phi^-_2 \sigma_t \right).$$

We found that our results did not depend on separating liquid vs. illiquid times, or the linear specification itself, with a similar data fit and model properties as before. For instance, the escalator–elevator effect holds with $\hat{\phi}^+_1$ less negative than $\hat{\phi}^-_1$ and $\hat{\phi}^+_2$ less positive than $\hat{\phi}^-_2$. In fact, the estimated price impacts $(c^+_t, c^-_t)$ and the corresponding liquidity returns have similar properties in this new model, being negligible for long periods of time.

Finally, we found an unbounded precision of private signals or low information disparities across strategic agents, which we considered a natural feature of FX markets. Moreover, even if we think of a common knowledge set–up and hence a situation of multiple equilibria, both the risk–dominant and payoff–dominant equilibria coincide with the (unbounded precision) global game equilibrium. Therefore, even in a common knowledge set–up, the model estimated in Table 2 represents the relevant equilibrium of our coordination game.

Nevertheless, it is interesting to compare the quantitative implications of this equilibrium to some other ad–hoc equilibrium. Given the estimates of Table 2, fundamental returns fall into the multiple equilibria region around 84% of the periods with liquidity tensions. We chose a simple ad–hoc equilibrium where the strategic agents aggregate behavior is like a coin toss in the multiple equilibria region, i.e. we randomize $\lambda^e_t = 0$ or 1 with probability 0.5 in that region. Note that this represents an extra source of randomness in the game outcome.

The parameter estimates and the corresponding model properties change in the expected direction. Now liquidity returns absorb part of the role of fundamental returns in generating volatility clustering, and hence we find a lower $\hat{\gamma}_2$ with respect to the global game. At the same time, liquidity returns show lower skewness and kurtosis, due to $(\hat{\phi}^+_1, \hat{\phi}^+_2)$ being similar.
to the global game but $\left( \hat{\phi}_1, \hat{\phi}_2 \right)$ being less extreme. That is, the tail risk is a bit less pronounced in the new game. But in general, the quantitatively implications for observed returns are similar to the global game, in fact the data fit is similar.

The main difference between both games is how they decompose similar total return properties into different fundamental and liquidity returns. They are close to being uncorrelated in the ad–hoc equilibrium, with a correlation of 2%. Given the new estimated parameters, fundamental returns fall into the multiple equilibria region 78% of the time, when liquidity tensions and outcomes are random and hence unrelated to fundamental returns. Similarly, we can translate the main difference between both equilibria to the correlation between fundamental returns and $\lambda_t$ itself. The estimated correlation is -9%, much lower in absolute value than in the global game.

The properties of fundamental and liquidity returns in the global game are closer to the empirical literature that shows that financial crises are related to fundamentals. To test if the global game correlation between $\lambda_t$ and fundamental returns is plausible we would need data on traders’ actions, i.e. quantity or order data. At the daily frequency it is difficult to find public quantity data in currency markets, and even when they are available at lower frequencies they may not be reliable enough.\textsuperscript{15}

5 Conclusions and Further Research

This paper represents a first attempt to the integration of global games and empirical finance. Using the efficient method of moments and only publicly available data, we showed that a simple coordination model fits well a long span of daily data that covers the yen-dollar carry trade turmoil in 1998. The empirical results are robust, both in terms of proxies and especially specifications. Our robustness checks indicate that what we are really capturing is the strategic behavior, and we are convinced that the results are not simply due to the econometric structure of fundamentals and liquidity.

In particular, we found the yen–dollar market to be generally very deep but also subject to clear up by the escalator–down by the elevator episodes. The\textsuperscript{15}For instance, at the quarterly frequency, Brunnermeier et al. (2008) run some regressions to explore the impact of carry trades and use non-commercial futures positions to proxy speculative positions. This proxy is quite noisy and hence the authors find difficult to obtain significant results with it. This is not surprising because the non-commercial classification may be imperfect to detect speculative positions and moreover the important OTC positions are outside this proxy.
asymmetry between positive trends and sharp corrections in prices is due to liquidity tensions on the sell side being much less frequent but also much more sensitive to risk appetite. The market fragility itself is due to low information disparities amongst the game players.

On the other hand there are some interesting issues that are outside the scope of this paper and hence are left for future research. We could apply our methodology to the estimation of new models that enrich the information structure and the dynamic behavior of strategic agents. The informational content of prices in global games has recently been studied by Angeletos and Werning (2006), Hellwig et al. (2006), and Tarashev (2003). Similarly, richer dynamic behavior in global games, e.g. learning across several trading rounds, is considered by Angeletos et al. (2007), and Dasgupta (2007). Abreu and Brunnermeier (2003) and specially Plantin and Shin (2007) are other relevant references on dynamic behavior, close in spirit to global games.\footnote{The predatory trading model in Brunnermeier and Pedersen (2005), which represents a different type of coordination, could be another interesting application of our methodology.}

Furthermore, our focus has been on market risk but credit risk is another relevant application of coordination games, and hence another area where we could apply our methodology. In particular, Ordonez (2008) develops a (non-standard) dynamic global game that delivers clustering in risk-taking without a big change in fundamentals. His theoretical results are consistent with the descriptive empirical evidence in Das et al. (2007), where fundamentals do not seem enough to explain default clustering.

\section{Proof of Lemma 1}

The proof is developed in two stages. The first stage shows that the condition in Lemma 1 is a sufficient and necessary condition for a unique symmetric equilibrium. The second stage shows that the corresponding switching strategy is the only one that survives iterated dominance, i.e. there cannot be any other equilibrium when the symmetric one exists.

Given the previous set-up, it is natural to focus on monotone strategies. Let us focus also on a symmetric equilibrium where all agents sell if \( x_t < x^o_t \). By means of studying the marginal strategic agent, we can compute the Bayes-Nash equilibrium of this imperfect information game in a similar way to Morris and Shin (1998). The marginal agents’s signal must be such that \( E (r_t \mid \Omega^o_t) = 0 \), or equivalently

\[
E (v_t \mid \Omega^o_t) + c^+_t = (c^-_t + c^+_t) \Pr (x_t < x^o_t \mid \Omega^o_t),
\]
where we have assumed a law of large numbers can be applied to the cross section of signals given \( v_t \).

The above expressions require the updated beliefs of strategic agent \( o \) after observing her signal, regarding both the fundamental asset return and her inference about other agents’ signals. A bit of algebra shows that

\[
v_t \mid \Omega^o_t \sim N \left( m^o_t, (\alpha_t + \beta_t)^{-1} \right), \quad m^o_t = \frac{\alpha_t m_t + \beta_t x^o_t}{\alpha_t + \beta_t},
\]

\[
x_t \mid \Omega^o_t \sim N \left( m^o_t, \omega_t^{-1} \right), \quad \omega_t = \frac{\beta_t (\alpha_t + \beta_t)}{\alpha_t + 2 \beta_t},
\]

and hence the marginal agent’s condition is

\[
m^0_t + c^+_t = (c^-_t + c^+_t) \Phi [\sqrt{\omega_t} (x^o_t - m^o_t)].
\]

This equation can be further reparameterized in terms of \( m^o_t \)

\[
m^o_t + c^+_t = (c^-_t + c^+_t) \Phi \left[ \sqrt{\omega_t} \frac{\alpha_t}{\beta_t} (m^o_t - m_t) \right],
\]

and then we find that a sufficient condition for a unique \( m^o_t \) is that the slope of the right hand side is lower than one everywhere, i.e.

\[
\frac{\alpha^2_t}{\beta_t (\alpha_t + 2 \beta_t)} \leq \frac{2\pi}{(c^-_t + c^+_t)^2}.
\]

This condition is also necessary in the sense that if it does not hold then we can choose an \( m_t \) such that there will be multiple \( m^o_t \) solving the previous equality.

To sum up, if that inequality is satisfied then we know that there is a unique equilibrium such that every strategic agent buys the asset if and only if her signal is such that \( x_t < x^o_t \), where \( x^o_t \) is given by

\[
x^o_t = \beta_t^{-1} \left[ (\alpha_t + \beta_t) m^o_t - \alpha_t m_t \right]
\]

with \( m^o_t \) solving the previous fixed point equation.

The second stage of the proof follows a standard argument in the global games literature and hence we will briefly summarize it. We define \( u_t(\tilde{m}, \hat{m}) \) as the expected utility of a strategic agent that observes a signal such that her posterior mean about \( v_t \) is \( \tilde{m} \) and thinks that the rest of strategic agents sell if they observe a signal that implies a posterior mean lower than \( \hat{m} \), i.e.

\[
u_t(\tilde{m}, \hat{m}) = \tilde{m} + c^+_t - (c^-_t + c^+_t) \Phi \left[ \sqrt{\omega_t} \frac{1}{\beta_t} \left( (\alpha_t + \beta_t) \hat{m} - \beta_t \hat{m} - \alpha_t m_t \right) \right].
\]
We can show that \( u(t, \hat{m}, \tilde{m}) \) is increasing in \( \tilde{m} \) and decreasing in \( \hat{m} \). This feature allows the construction of a sequence of increasing values of \( m \) and another of decreasing values of \( m \) that imply the deletion of many strategies by iterated dominance arguments. Both sequences converge to the solution of \( u(t, m^i_t, m^o_t) = 0 \), which is exactly the unique symmetric equilibrium that we constructed before. That is the only strategy that survives iterated dominance.

Finally, we can show the implications for \( r_t \) in equilibrium. For each realization of \( v_t \),

\[
x_t \mid \Omega^o_t, v_t \sim N(v_t, \beta_t^{-1})
\]

and hence

\[
\lambda_t^e = \Pr(x_t < x_t^o \mid \Omega^o_t, v_t) = \Phi\left(\sqrt{\beta_t} (x_t^o - v_t)\right).
\]
References


ORDONEZ, G. L., “Fragility of Reputation and Clustering in Risk Taking,” (2008), mimeo, UCLA.


